Stability & Noise in Active Metamaterials

Khalid Z. Rajab, Yifeng Fan, Deepak S. Nagarkoti & Yang Hao

Slide	
1	In this presentation we discuss using active non-Foster circuits (NFC) for loading structures such as electrically-small antennas (ESA) and metamaterials. This is to reduce losses and increase bandwidth.
2	Start with a discussion and history of active structures, including ESAs and metamaterials. We will then discuss the two key issues: stability and noise.
3	Non-Foster loads have been used for broadband matching of ESAs. There are studies dating back to 1962, where antennas were matching in the kHz range.
4	What are the motivations for using active metamaterials? The references cited show that causality limits the bandwidth of passive materials with extraordinary properties. They are also necessarily lossy. This means that structures such as the free space invisibility cloak has a narrow bandwidth. This is demonstrated with the animations, where the cylinder is successfully cloaked from red light, while blue light leaves a shadow.
5	Various active schemes have been proposed for loading metamaterials: phase-shifters at microwave frequencies, and gain media for optics. There have been a number of studies of non-Foster loads for increasing the bandwidth of active metamaterials.
6	A key concept of the NFC is that, for a lossless load the slope of the reactance is always positive with frequency. A negative impedance circuit (NIC) is very commonly used to implement the NFC. These date back to vacuum tube models from the 1930s. Linvill's transistorised version, invented in 1953, is the most popular model, and was widely used as signal boosters in the trans-Atlantic telephony cables.
7	How do NICs work? The current direction is reversed at the input, compared to at the load, while the voltage polarity remains the same. Alternatively in a different implementation, the voltage polarity may be reversed while the current direction is the same.
8	We have built stable NICs at Queen Mary University of London. Here we show extracted parameters of various fabricated negative capacitors.
9	Previously we have demonstrated that stable negative permeability (μ) materials may be implemented. These investigations took into account all mutual coupling and periodicity effects. It has also been shown by Hughes Research Lab (HRL) that stable wideband artificial magnetic conductors (AMC) can be fabricated.
10	Stability is a primary concern of these active structures. Here we demonstrate measurements of unstable negative capacitors. These were measured using a vector network analyser. Similarly, they may be measured by viewing emissions peaks with a spectrum analyser.
11	The capacitors are made stable by adding a positive capacitance to cancel its negative counterpart. In the next slides we will examine how to determine stability. It is important to note that we cannot use the classic amplifier stability equations (μ or K-Δ tests). These tests rely on the assumption that there are no poles in the right-hand side of the complex plane. However with the NFC this may easily be the case, leading to instability.
12	The importance of stability is discussed here. This may be examined analytically by using the Routh-Hurwitz criterion.
13	Results of the Routh-Hurwitz criterion applied to a bulk magnetic material are shown here. The analysis is from the paper cited. The results have been confirmed using FDTD simulations.
14	A better technique for analysing stability is the Nyquist criterion. We show that for a Thevenin equivalent circuit with a source and load, the voltage v can be expressed in terms of the

	numerator and denominator coefficients. Defining N_l and D_s such that they have no RHP zeros, then the closed-loop system is stable if $(1+Z_sY_l)$ has no zeros.
15	We define a gain function G, and using Nyquist's stability criterion we can state that the system is stable if the G does not circle the -1 point in the complex plane. The NFC is defined as either the load or the source, depending on whether it is open-circuit stable (OCS) or short-circuit stable (SCS). A simple case is demonstrated, of a positive resistor and inductor in series with a negative inductor. Clearly, if the overall inductance is negative then the system is stable. This is confirmed by the loop encircling the -1 point.
16	Another example is shown, to demonstrate the importance of parasitics. While the system is clearly unstable, a loop is not evident that encircles the -1 point. Because the gain G is non-zero at DC ($s\rightarrow 0$), this suggests that we will need to investigate the parasitics in more detail.
17	In this scenario the circuit is almost identical to the previous case, but with the addition of a negative conductance in parallel with the negative capacitor. This parasitic conductance may be infinitesimally small, but it will set the DC gain to 0, and reveals an additional loop that was not accounted for previously. This loops demonstrates that the system is unstable. This case shows that it is important to account for parasitics in the stability modelling, or it will be easy to miss loops that will imply instability.
18	The Nyquist stability criterion is applied to an active magnetic metamaterial. The results correspond to the predictions of the analytical formulae derived using the Routh-Hurwitz technique. However the Nyquist method is more powerful as it can easily be analysed using realistic models, including parasitics and with measured data.
19	Results are confirmed using FDTD simulations. FDTD is a causal technique, and so it can reveal instabilities in the time-domain.
20	These techniques can be used to design a stable actively-loaded ESA, in this case a wire monopole. Bandwidth is shown to be significantly increased.
21	Noise is an important characteristic that must be detailed in any active system. We demonstrate our analysis of Johnson-Nyquist noise, using a noise matrix technique. Details are provided in the citation.
22	We have analysed noise in 0D (single loop), 1D (linear), 2D and 3D arrays. It is important to note that noise increases significantly when the material parameters become more extreme.
23	In conclusion we have shown that stable design is possible for active antennas and metamaterials. However parasitics are an important factor that must be accounted for in the design. Similarly, Johnson-Nyquist noise can have a detrimental effect on performance. There are important questions remaining: • Is the increase in bandwidth and gain worth it at the expense of possible instability and noise? • Can operational frequency of NFCs be increased? Current state-of-the-art NFCs operate at up to around 1GHz. Above this, parasitics are a serious problem. • Can NFCs operate with high powers? When used for broadband matching of ESAs, there is high reactive power stored in the NFC. This means that currently, they can only be used for low-power applications, i.e. receive. A new NFC design is necessary to overcome this performance issue.

School of Electronic Engineering and Computer Science

Stability & Noise in Active Metamaterials

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University of London

Overview

- Non-Foster loaded structures
- Stability analysis
  - Overview of techniques
  - Examples
- Noise
  - Non-Foster circuit
  - Active metamaterials
- Conclusions

Some History

AN INVESTIGATION OF BROADBAND MINIATURE ANTENNAS

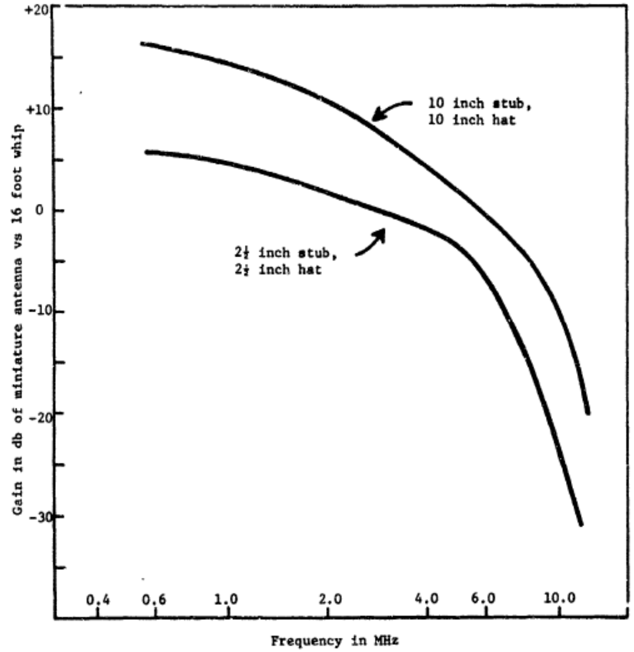
by

Andrew Durham Harris
Captain, United States Marine Corps

and

Glen A. Myers
Associate Professor of Electrical Engineering
Naval Postgraduate School
Monterey, California

September 1968



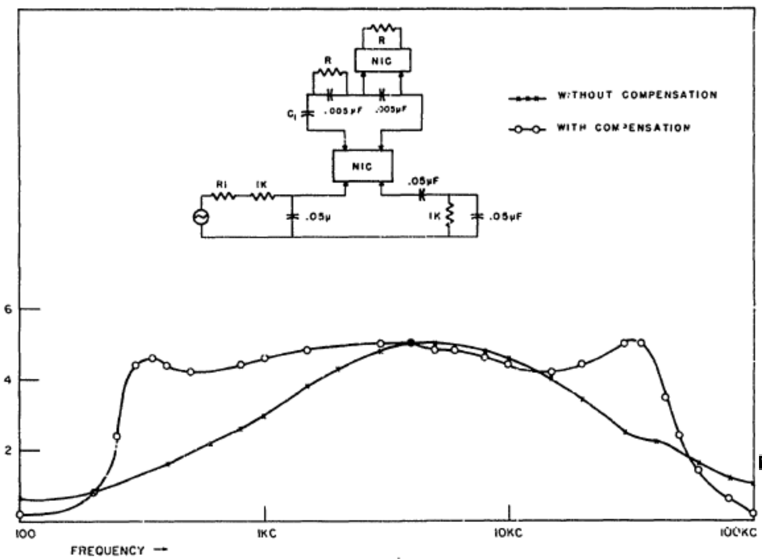


Figure 26. Compensation Using Two NIC s.

Antenna Laboratory

Technical Report No. 64

ANTENNA IMPEDANCE MATCHING
BY MEANS OF
ACTIVE NETWORKS

by

S. Laxpati
R. Mittra

Contract AF33(657)-8460
Hitch Element Nr. 62405454
760D - Project 4028, Task 402824
Aeronautical Systems Division
Project Engineer E. Turner, ASRNCF-1

November 1962

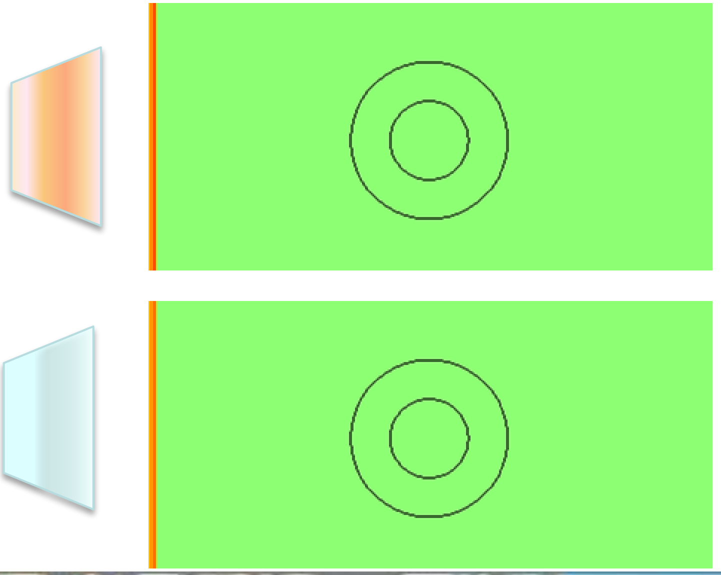
Aeronautical Systems Division
Wright-Patterson Air Force Base, Ohio

ELECTRICAL ENGINEERING RESEARCH LABORATORY
ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS

Why Active Metamaterials?

- High dispersion of passives
- Limitations due to causality/passivity:
  - MI Stockman PRL **98** 177404 (2007)
  - B Nistad and J Skaar PRE **78** 036603 (2008)
  - M Gustafsson META '13 (2013)

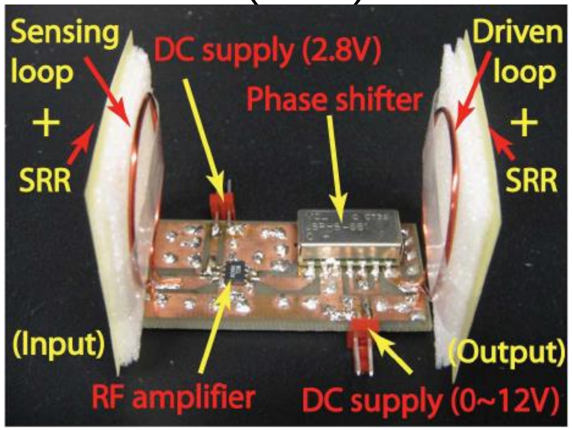
Class	Description	Limitation
ENG or ENZ (Wire medium, plasmon, etc.)	$\varepsilon <\approx 0$. Normally Drude-type dispersion.	$\varepsilon < 0$ over a large band, but highly dispersive.
MNG or MNZ (SRR, etc.)	$\mu <\approx 0$. Normally Lorentz-type dispersion.	$\mu < 0$ over a very narrow band at resonance. High losses.
TL-based	Left-handed transmission line, with series C and shunt L.	Left-handed over a large band, but highly dispersive ε_{eff} and μ_{eff}.



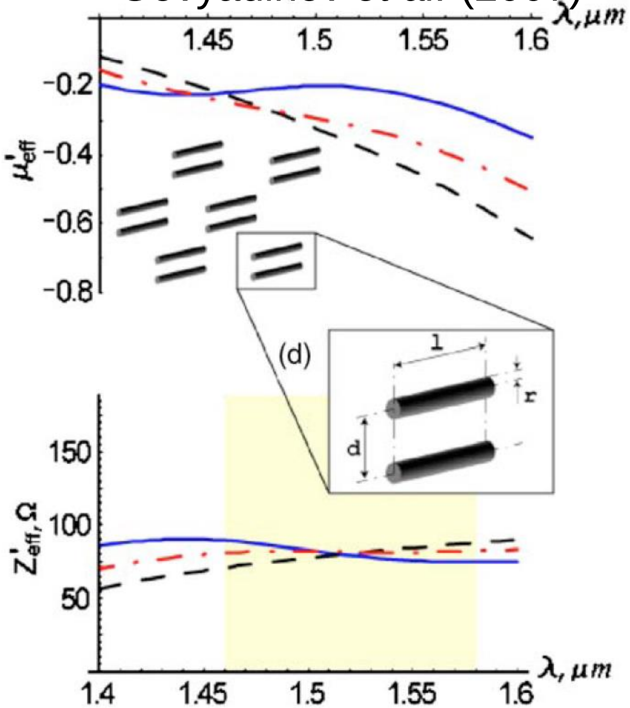
Actively-loaded metamaterials

- Motivation for active loads:
  - Increase bandwidth of:
    - Bulk materials (NG and NZ).
    - TL-based
- Active bulk metamaterials:
  - Active non-Foster loads:
    - Broadband ES antennas (Bahr: IEEE TAP 1977; Sussman-Fort: IEEE TAP 2009).
    - Proposed by Tretyakov (MOTL 2001).
    - ENZ (Hrabar: Meta 2009/2010, APS 2010).
    - MNZ/MNG (Rajab: JAP 2010, APS 2010, Meta 2010).
  - Other active schemes:
    - Microwave: phase shifter (Yuan: Opt. Exp. 2009).
    - Optical: Active gain(Govyadinov: APL 2007).

Yuan *et al.* (2009)

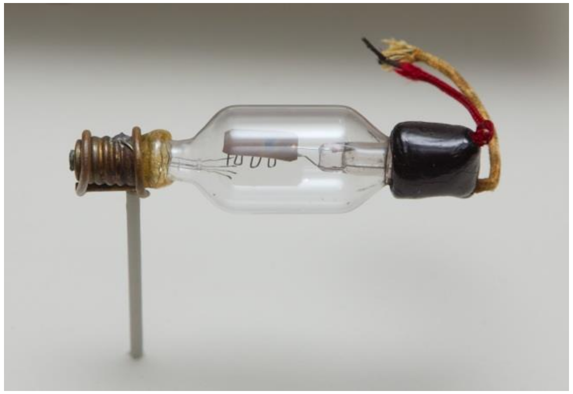


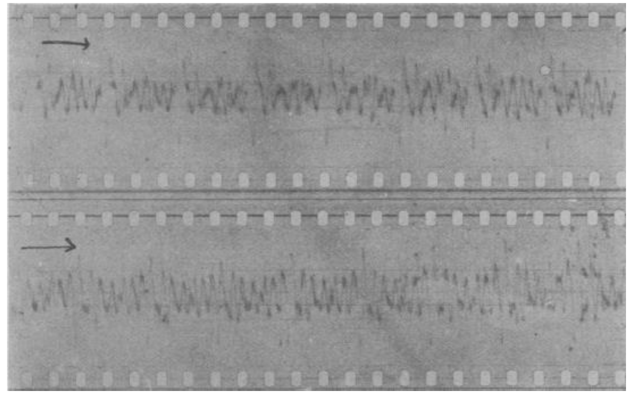
Govyadinov *et al.* (2007)

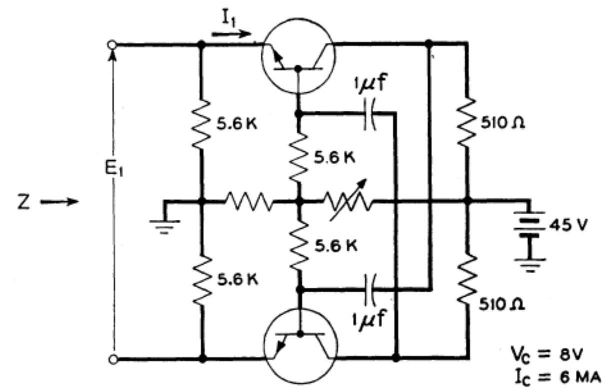


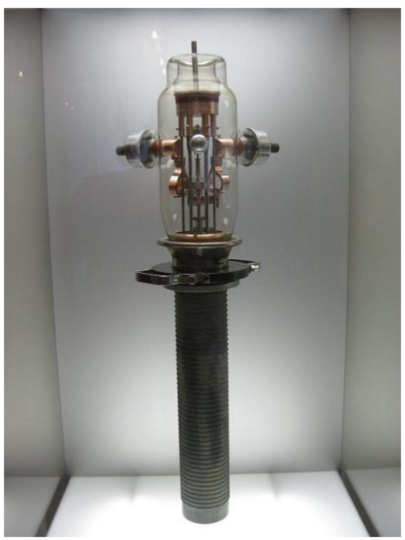
The Negative Impedance Converter

'A lossless reactance always has a positive slope of reactance with frequency.' Foster (1924).

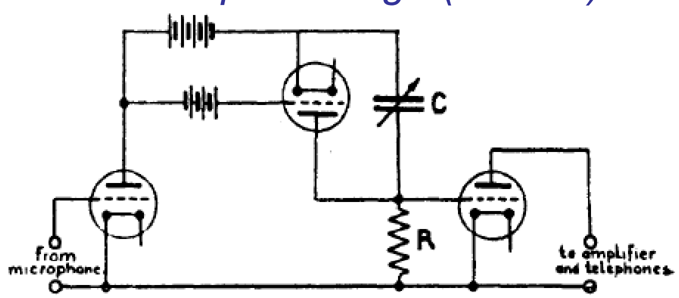


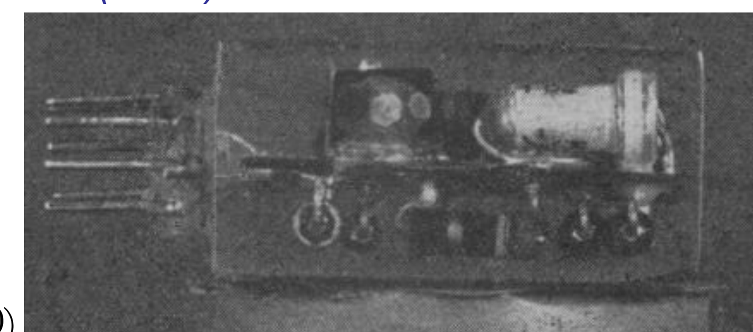






'The ideal NIC is an active four-pole with input current (voltage) equal to output current (voltage), and input voltage (current) equal to the negative of the output voltage (current).' Linville (1953)



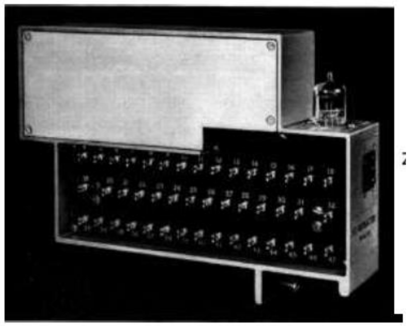


B Van der Pol Proc. IRE **18** 220–30 (1930)

JG Linvill Proc. IRE **41** 725–9 (1953)

How?
Negative Impedance Circuit design

Bell Telephone E2 Repeater

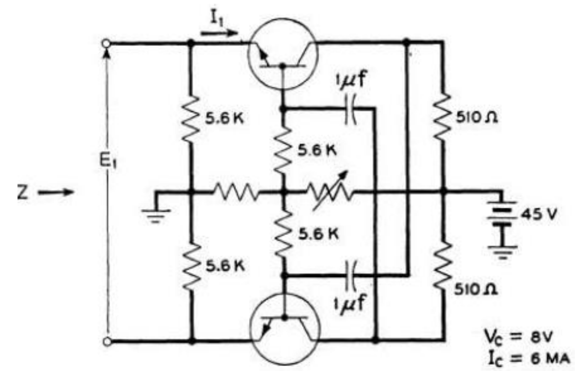


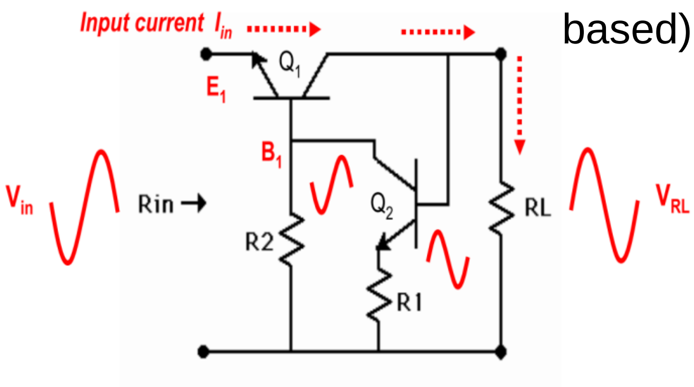
- Linvill 1953, Bell Labs

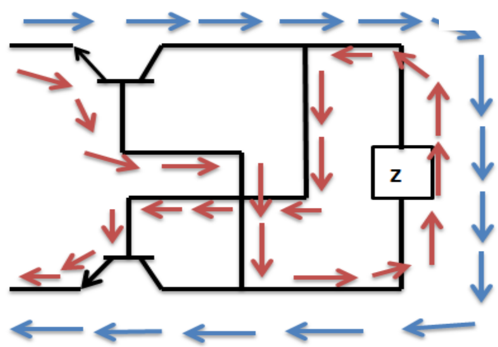
Motivation- Gain in telephone repeater

- Transistor based –Robust and light (not fragile and bulk like vacuum tube based)

Linvill Transistor-Based NIC



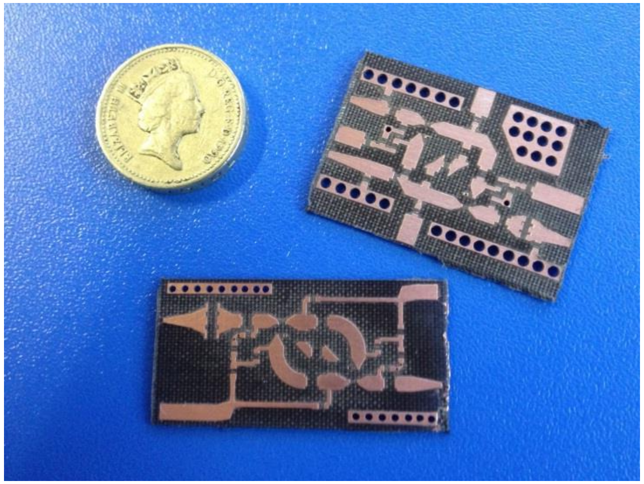


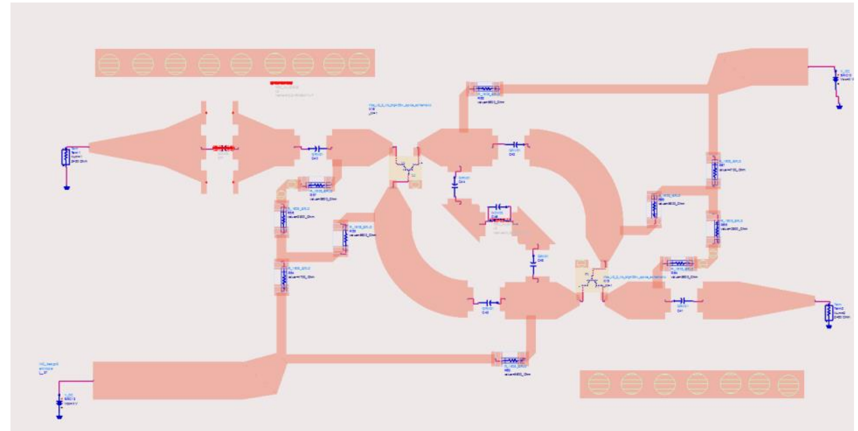


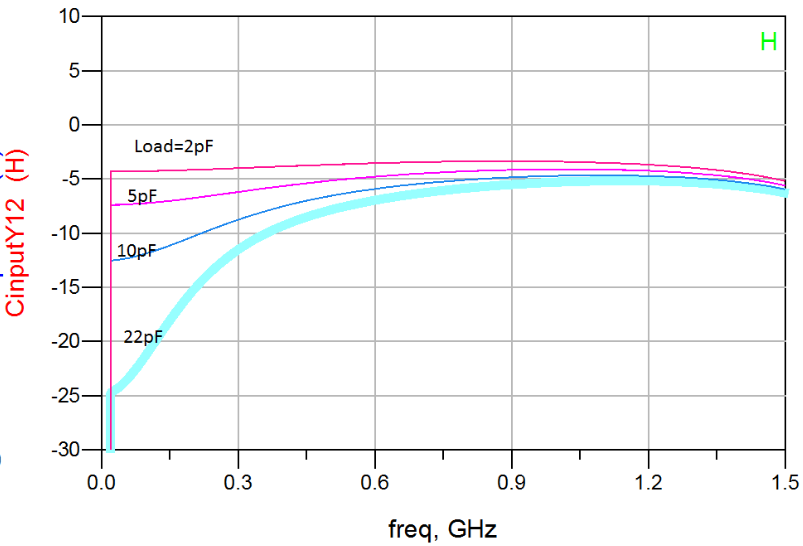
- Current and voltage in same direction at load but voltage reversal at input.

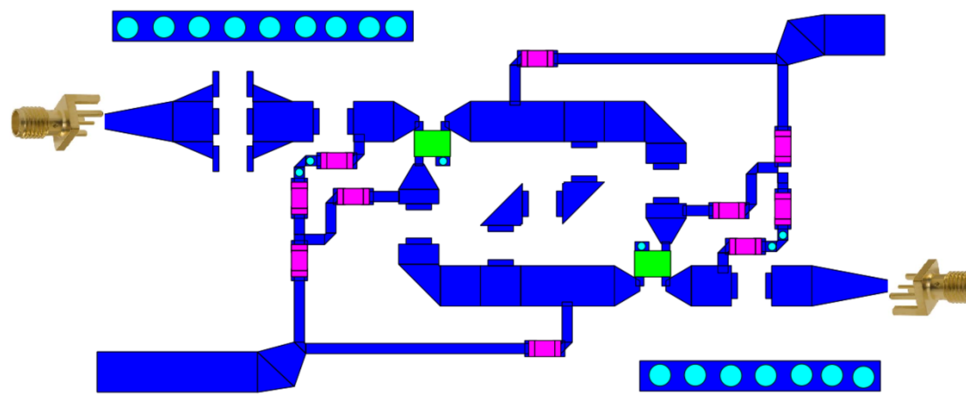
- Current and voltage in opposite direction at load but same at input.

NIC Circuits









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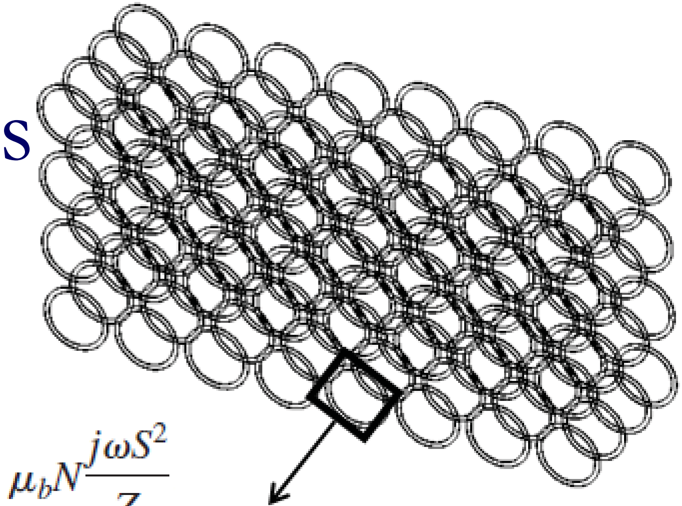
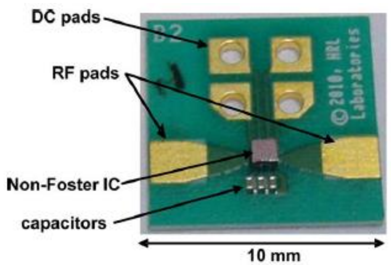
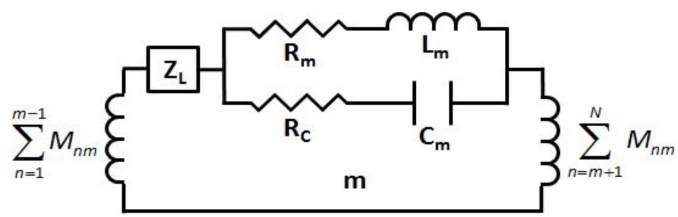
Active Metamaterial Properties

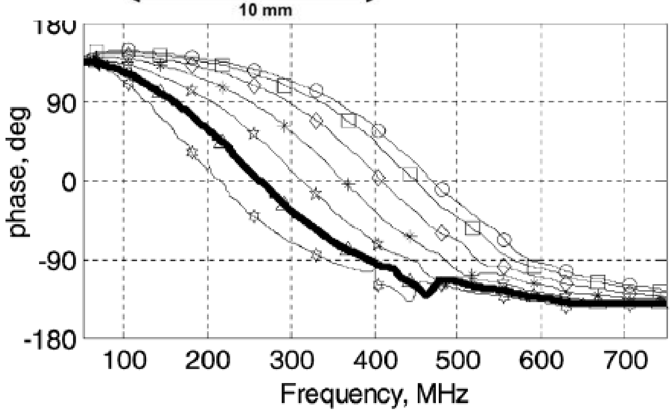


- Bulk material (or surface) loaded with NICs
- KZ Rajab *et al* JAP **108** 054904 (2010)
- KZ Rajab *et al* J Opt **14** 114004 (2012)

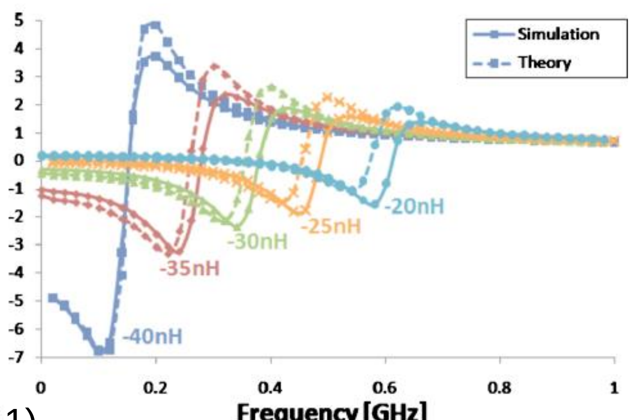
$$\mu_r = 1 - \mu_b N \frac{j\omega S^2}{Z}$$

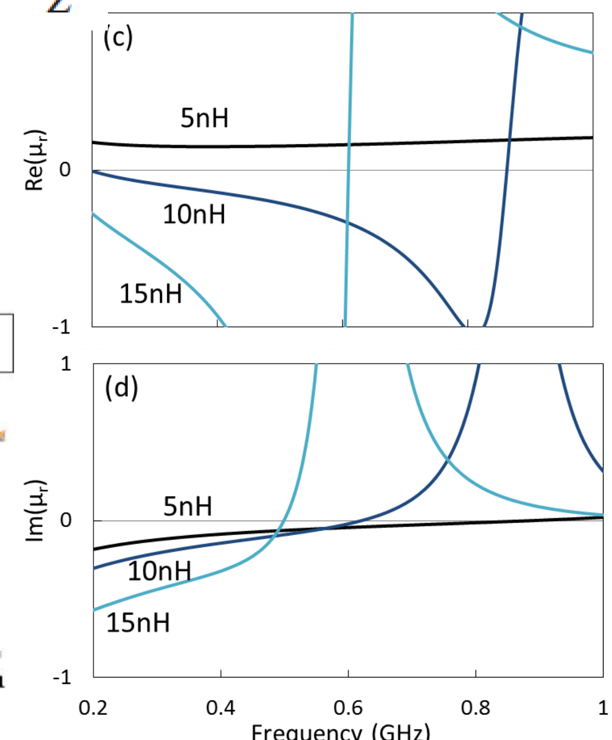




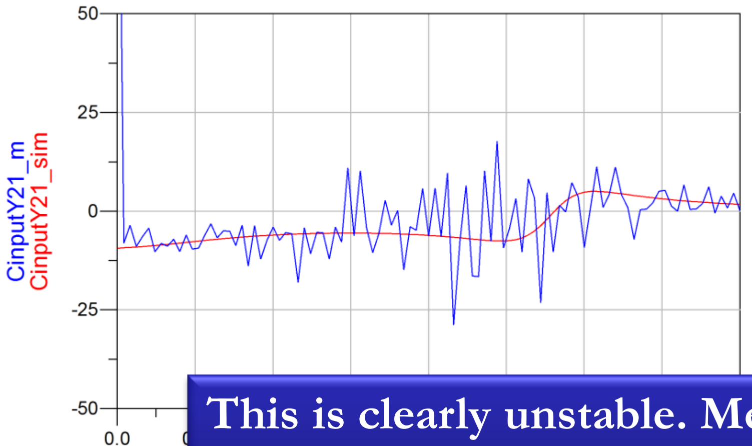


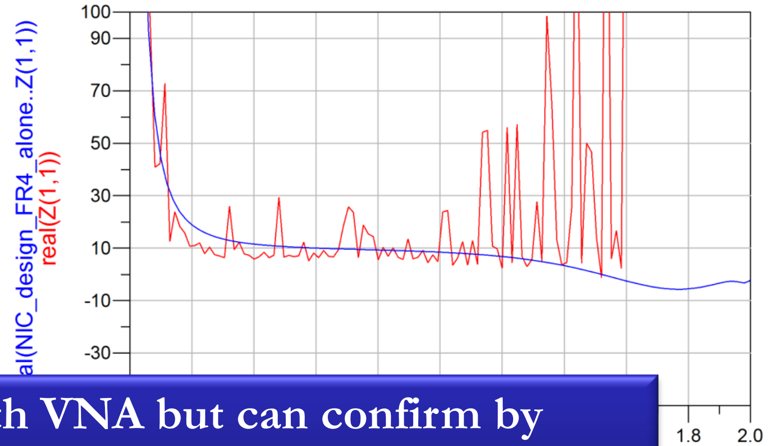
DJ Gregoire *et al* IEEE AWPL **10** (2011)



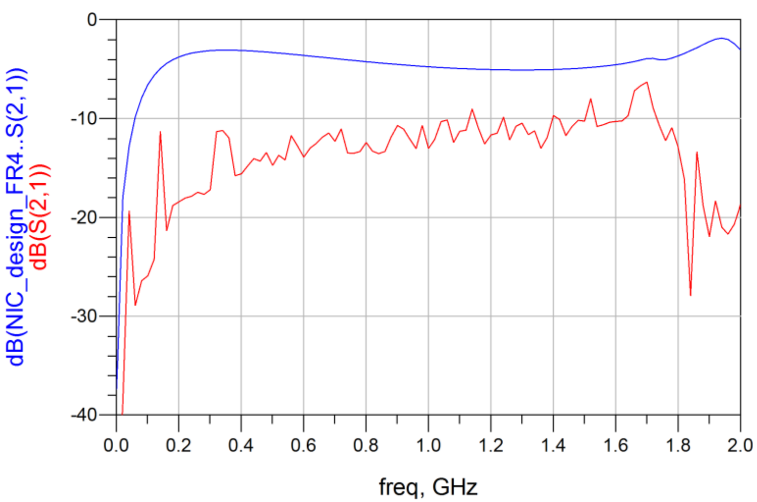


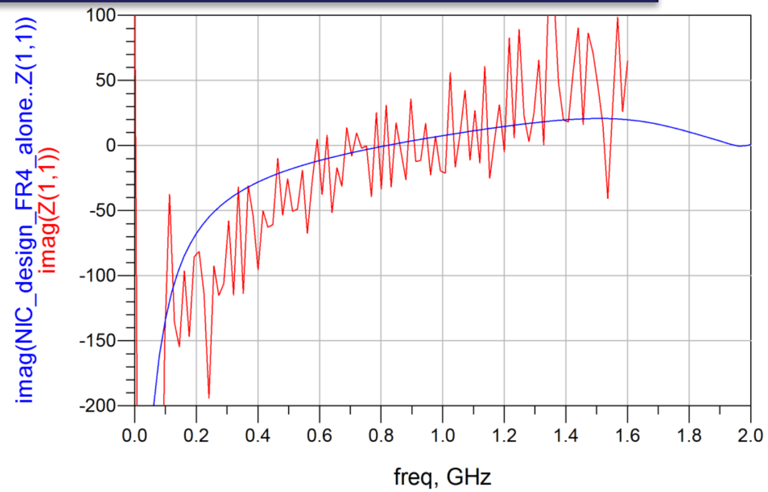
Unstable –C!



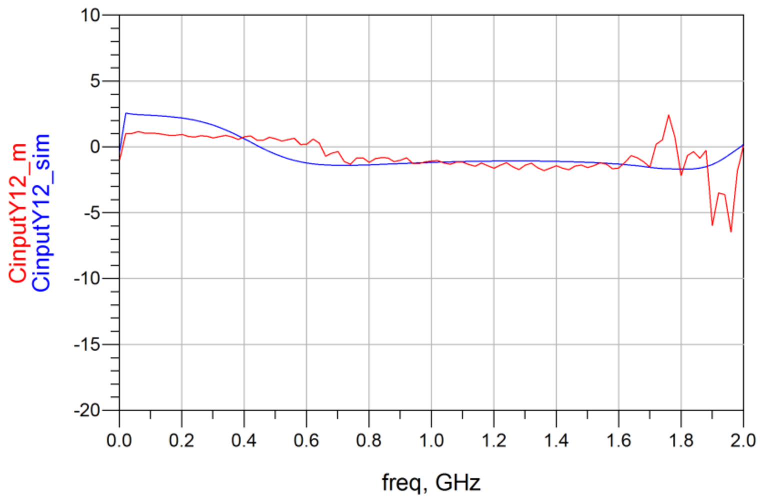


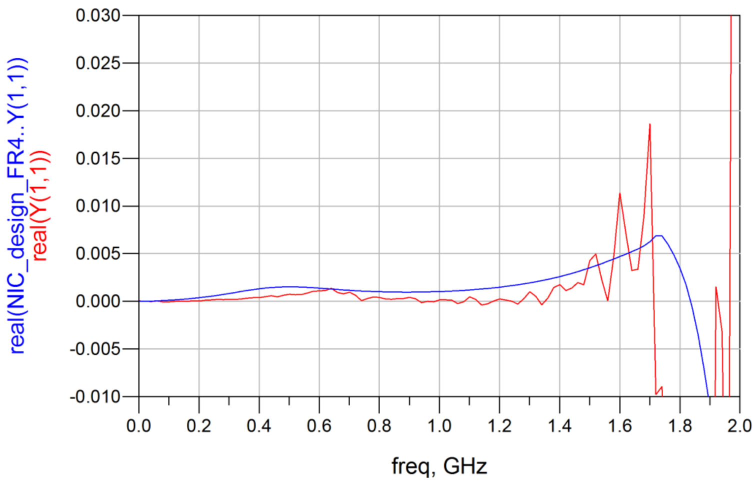
This is clearly unstable. Measured with VNA but can confirm by measuring emission peaks with Spectrum Analyzer.

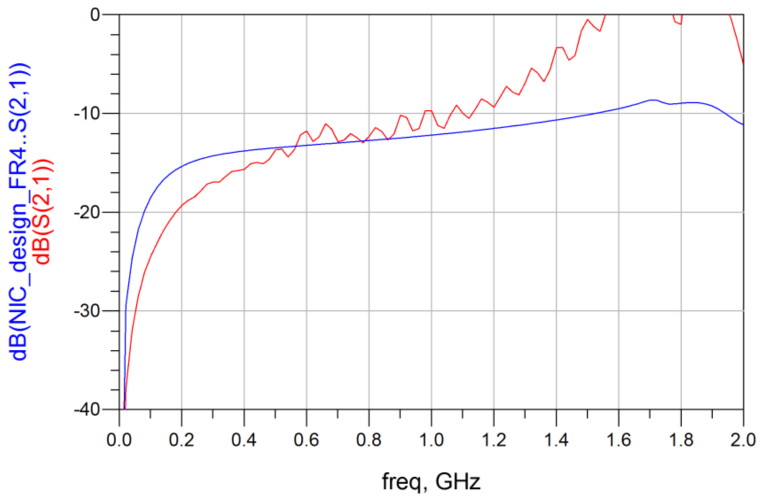




Stable -C







Add a 1pF capacitor to make it stable.

How do we design for stability?

We cannot use Amplifier stability equations!

Stability analysis

- Why is it important?
  - The use of active components can lead to instability
  - This becomes even more important when mutual coupling is present.
- Analysed using Routh-Hurwitz criterion.
  - Characteristic polynomial: $$Z_n(s) + s\lambda_M Z_d(s) = \sum_{k=0}^{K} p_k s^k = 0$$
  - Create a Routh array: $$\bar{\mathbf{R}} = \begin{pmatrix} p_K & p_{K-2} & p_{K-4} & \cdots \\ p_{K-1} & p_{K-3} & p_{K-5} & \cdots \\ R_{31} & R_{32} & R_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$
  - Where: $$R_{mn} = \frac{1}{R_{m-1,1}} \begin{vmatrix} R_{m-1,1} & R_{m-1,n+1} \\ R_{m-2,1} & R_{m-2,n+1} \end{vmatrix}$$
  - All roots are in LHS of complex plane (i.e. negative exponents →stability) if all coefficients in first column have the same sign.
  - Finite system has N eigenvalues to test. Infinite system has 1.

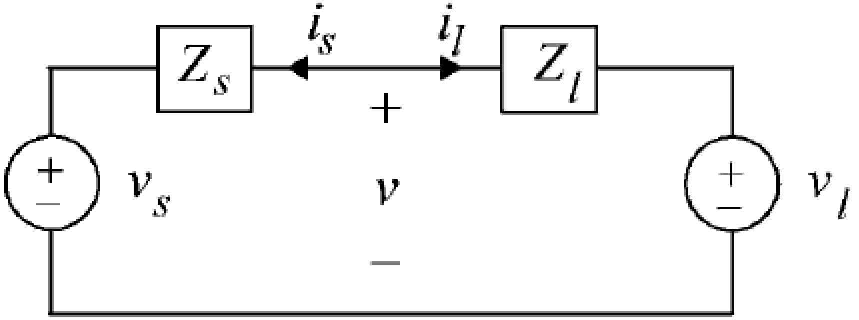
Stability criteria

- The load is a parallel RLC circuit.
- The following stability criteria have been derived:
  - Inductive load: $$0 > L_L > -\frac{R_L}{R_0 + R_L}\frac{[L_0 + \lambda_M]^2}{L_0 + \lambda_M + R_0 R_L C_L}$$
  - Inductive load (approximation for periodic case): $$0 > L_L \gtrsim -(L_0 + M_p)$$
  - Capacitive load: $$C_L < -\frac{L_0}{\omega_0(L_0 + M_p)}\sqrt{\frac{M_p(L_0 + M_p)}{|R_L|\left[(L_0 + M_p)^2 R_C + M_p^2 R_0\right]}}$$
- Interesting point:
  - Stability range decreases as number of loops N increases *but* stability range is maximized when system is infinitely periodic!
- We have provided an analytical solution. These are compared to numerical root-finding techniques, and FDTD simulations.

KZ Rajab *et al*, JAP **108** (2010) 054904

Stability – A Better Method

- $Z_S = \frac{N_S}{D_S}$
- $Z_\ell = \frac{N_\ell}{D_\ell}$

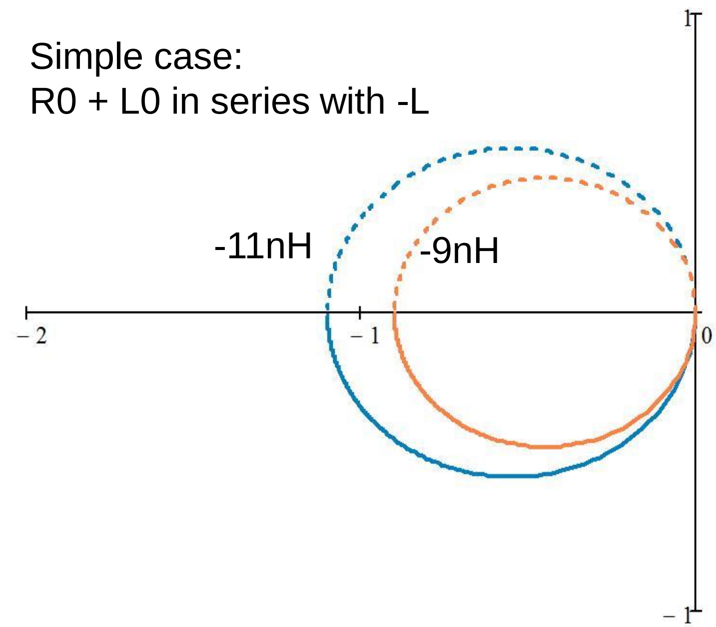


- Solve for v with a bit of rewriting:
- $v = \frac{N_\ell D_S v_S + N_S D_\ell v_\ell}{N_\ell D_S (1 + Z_S Y_\ell)}$
- No RHP zeros for N_ℓ and D_s
- **System is stable if $(1 + Z_S Y_\ell)$ has no RHP zeros**
  - **This is a closed-loop system**

Queen Mary University of London

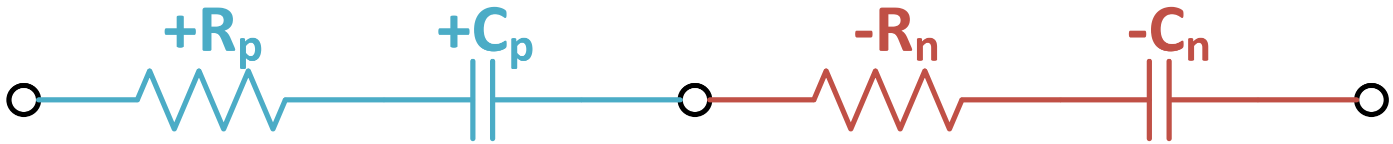
Nyquist Stability Criterion

- Define:
  - Gain function
    $$G(s) = Z_s(s)Y_\ell(s)$$
  - Cauchy's argument principle
    $$N = Z - P$$
  - Nyquist Stability criterion: **Can't circle -1 point! (clockwise)**
- Source or Load?
  - **OCS** Z poles in LHP → $Z_{NIC} = Z_S$
  - **SCS** Y poles in LHP → $Y_{NIC} = Y_\ell$



KZ Rajab *et al*, J Opt **14** (2012) 114004

Nyquist Stability Example 1



$$Z_n = \left(R_n + \frac{1}{sC_n}\right)^{-1}$$

$$Y_p = \left(R_p + \frac{1}{sC_p}\right)^{-1}$$

$$\boldsymbol{G = Z_n Y_p}$$

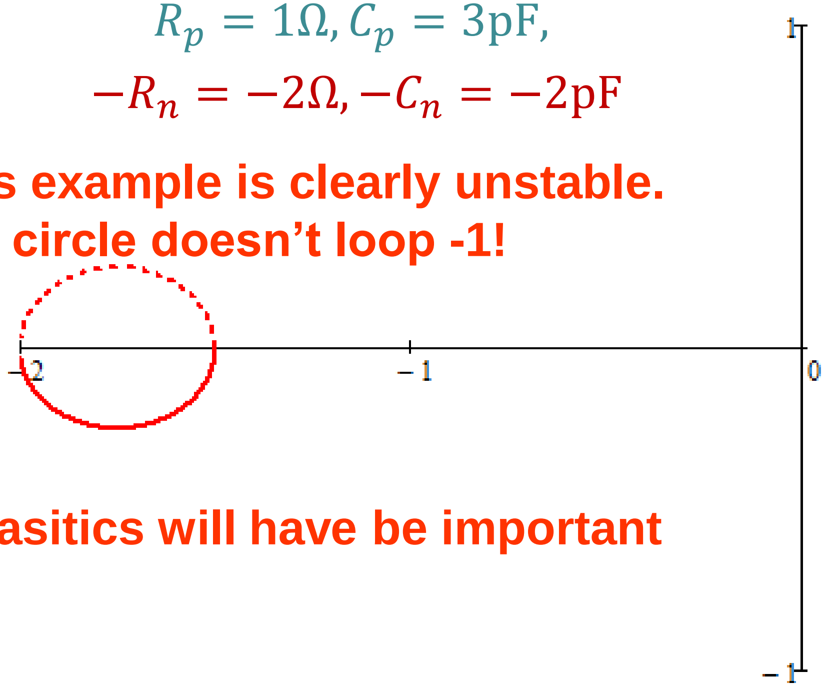
$$\boldsymbol{= -\left(R_n + \frac{1}{sC_n}\right) \cdot \left(R_p + \frac{1}{sC_p}\right)^{-1}}$$

$$\lim_{s \to 0} G \to -\frac{C_p}{C_n} \quad \lim_{s \to \infty} G \to -\frac{R_n}{R_p}$$

$$R_p = 1\Omega, C_p = 3\text{pF},$$

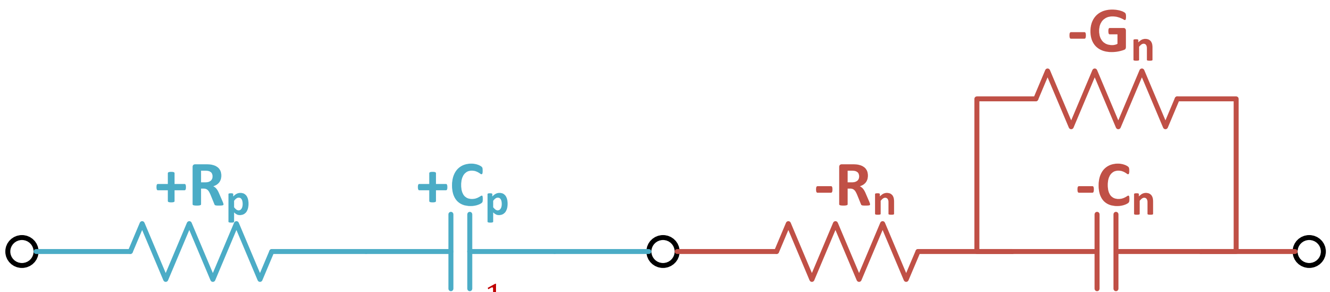
$$-R_n = -2\Omega, -C_n = -2\text{pF}$$

This example is clearly unstable.
But circle doesn't loop -1!



Parasitics will have be important

Nyquist Stability Example 2



$$Z_n = \left(R_n + \frac{1}{G_n + sC_n}\right)^{-1}$$

$$Y_p = \left(R_p + \frac{1}{sC_p}\right)^{-1}$$

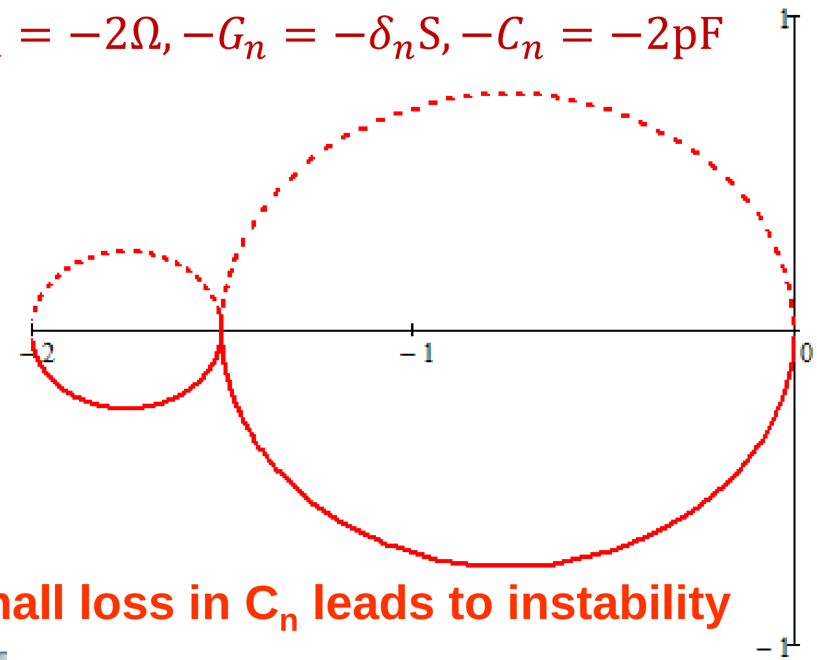
$R_p = 1\Omega, C_p = 3\text{pF},$

$-R_n = -2\Omega, -G_n = -\delta_n \text{S}, -C_n = -2\text{pF}$

$$\boldsymbol{G = Z_n Y_p}$$

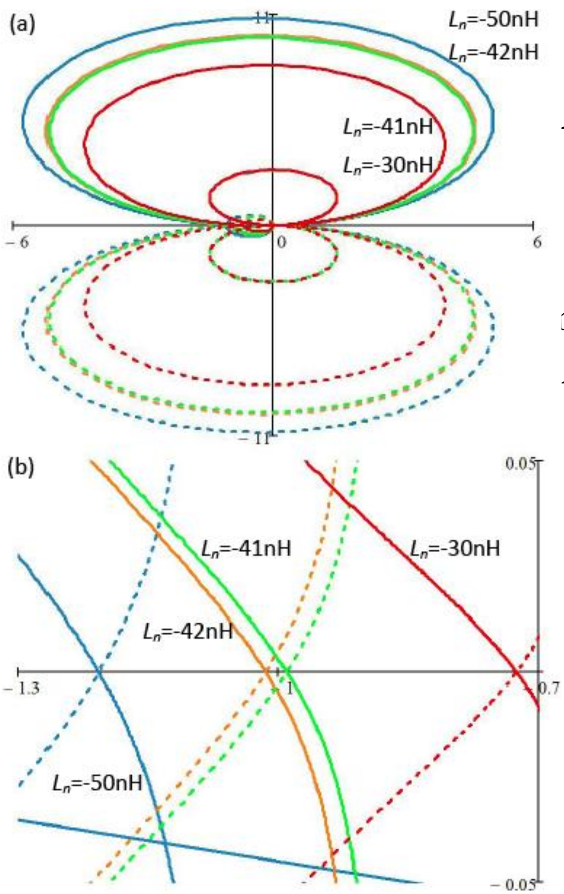
$$\boldsymbol{= -\left(R_n + \frac{1}{G_n + sC_n}\right) \cdot \left(R_p + \frac{1}{sC_p}\right)^{-1}}$$

$$\lim_{s \to 0} G \to 0 \quad \lim_{s \to \infty} G \to -\frac{R_n}{R_p}$$



A small loss in C_n leads to instability

Nyquist Stability Criterion



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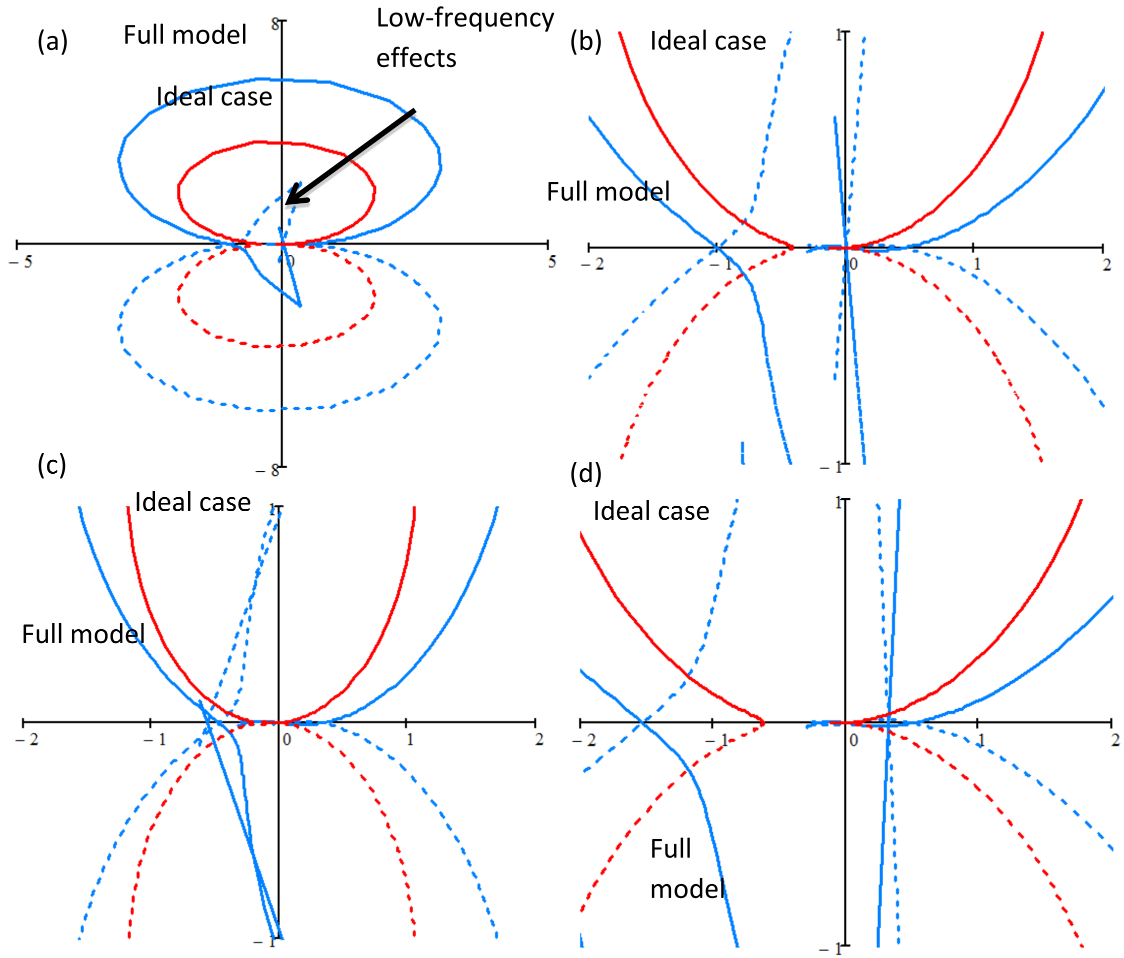
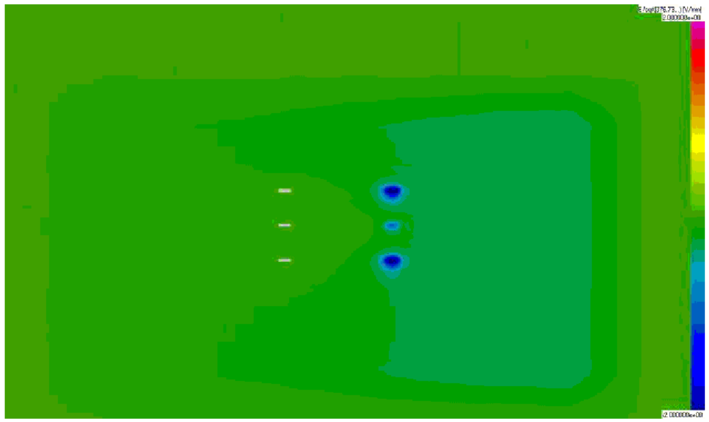
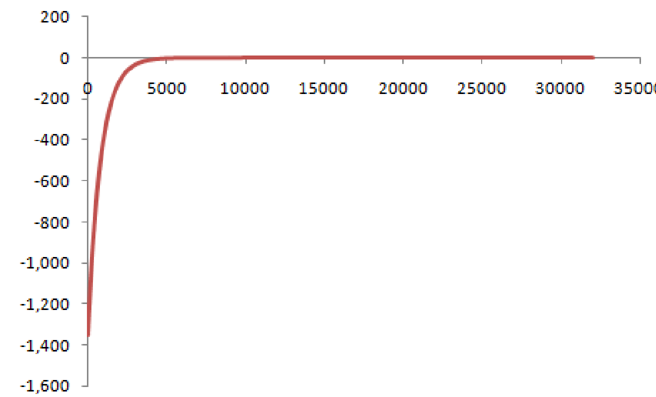


Stability verification with FDTD

Unstable





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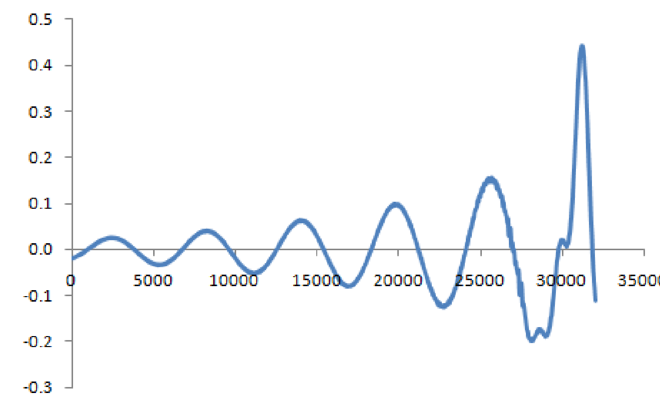
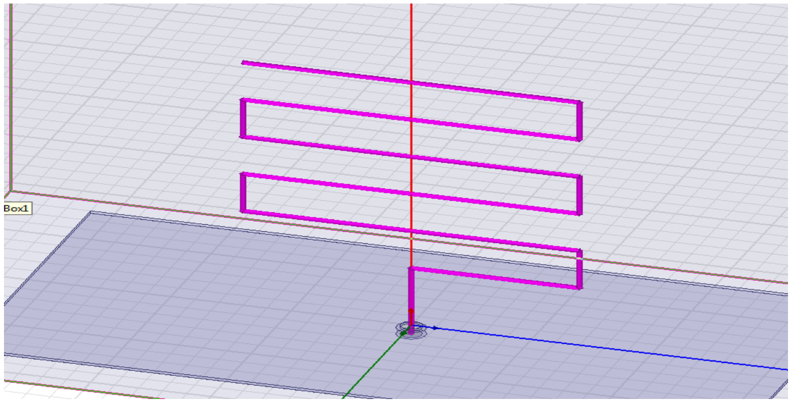
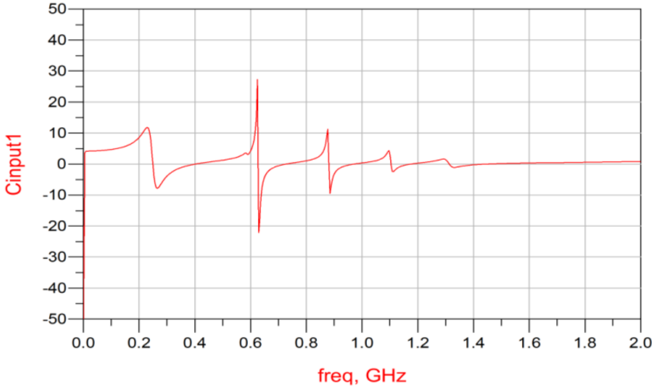
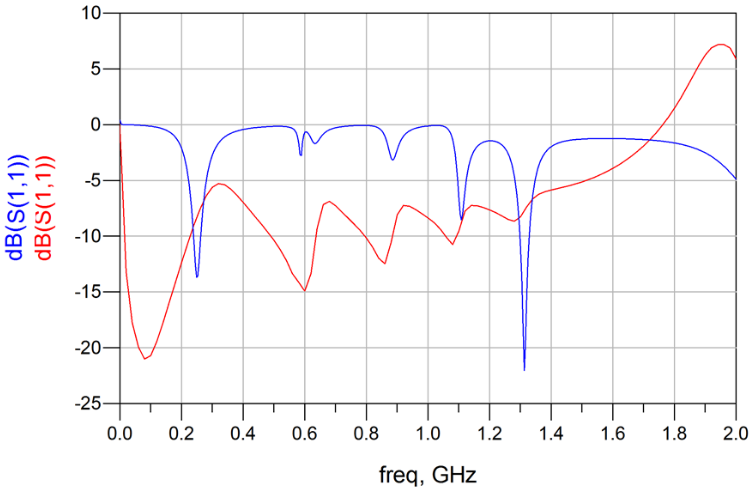
Stable

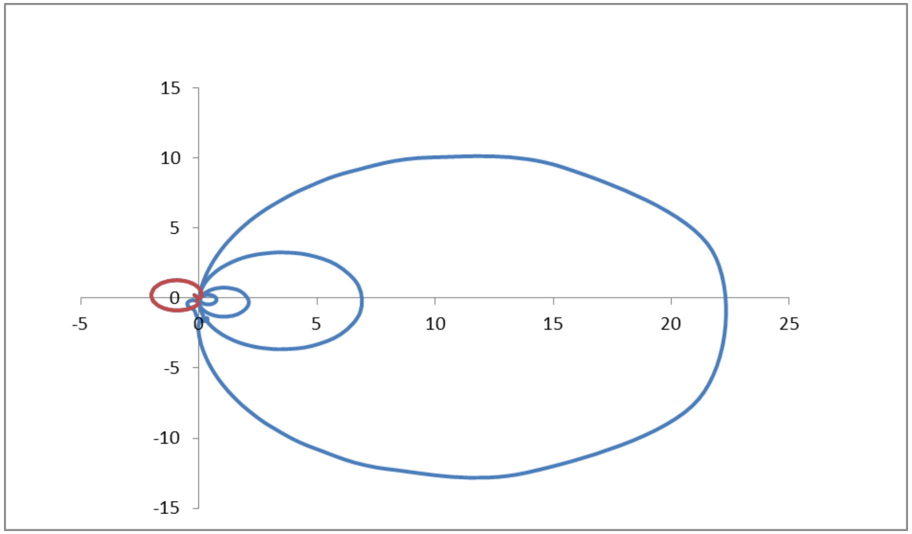


Antenna Matching

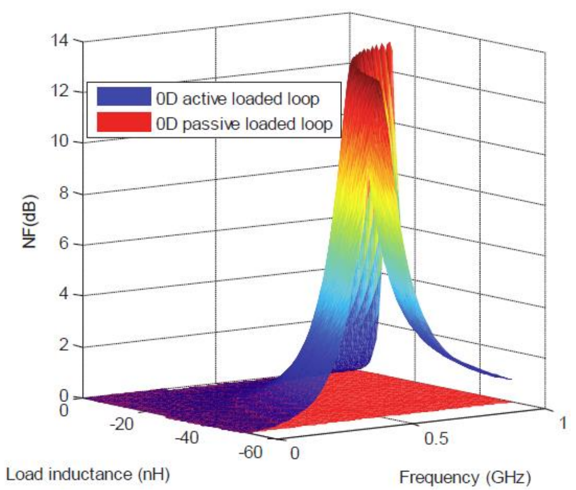




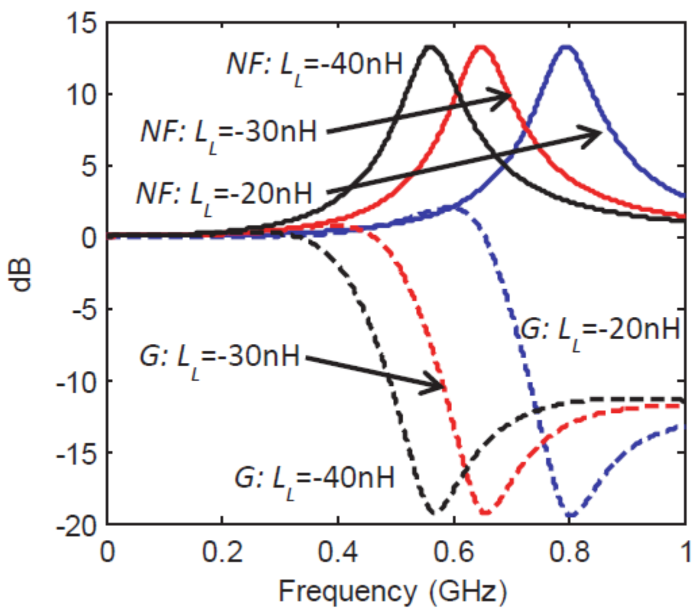




Noise in Active Metamaterials



- With active components, Johnson-Nyquist noise may be significant
- Will noise overwhelm potential benefits?
- Method:
  - Add noise source to every resistor
  - Matrix analysis including mutual coupling
- Reactive loads DO affect noise

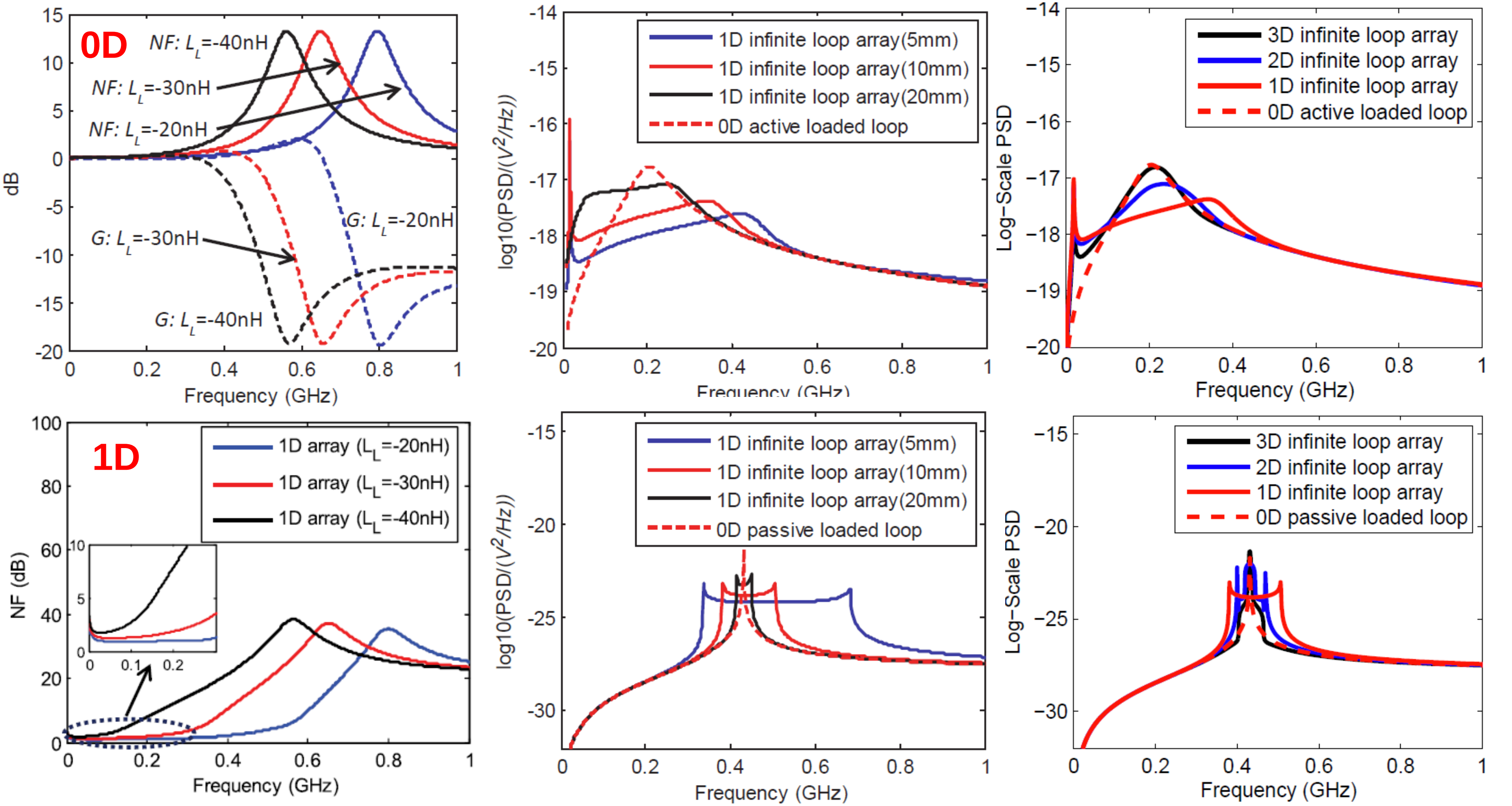


$$F_{0_{passive}} = 1 + \frac{|Z_{s2}|^2}{R_{source}} \frac{R_w |Z_{p3}|^3}{|Z_{s3}|^2 |Z_{L2}|^2}$$

$$F_{0_{active}} = \frac{S_i/N_i}{S_0/N_0} = 1 + \frac{N_{added}}{GKTdf} = 1 + \frac{|Z_{s2}|^2}{R_{source}} \left(\frac{R_w |Z_{p3}|^3}{|Z_{s3}|^2 |Z_{L2}|^2} + \frac{|Z_{p2}|^2}{|R_L || Z_{02}|^2} \right)$$

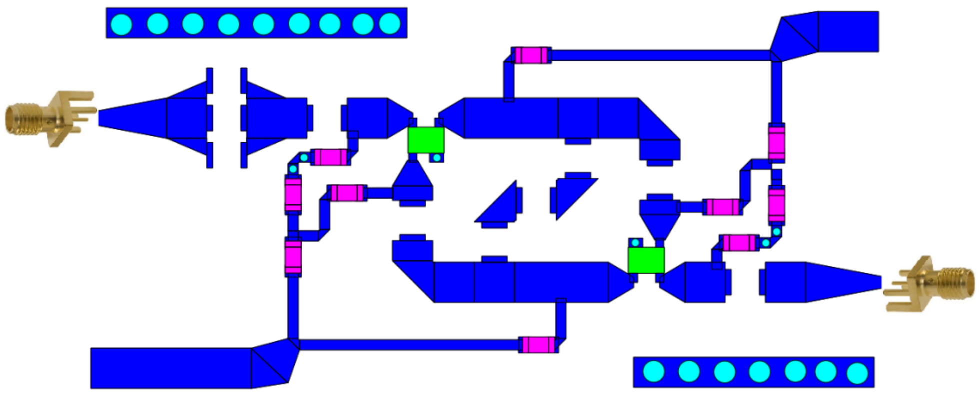
Y Fan *et al*, JAP **113** (2013) 233905

Noise



0D
NF: L_L=-40nH
NF: L_L=-30nH
NF: L_L=-20nH
G: L_L=-20nH
G: L_L=-30nH
G: L_L=-40nH
dB
Frequency (GHz)
1D infinite loop array(5mm)
1D infinite loop array(10mm)
1D infinite loop array(20mm)
0D active loaded loop
log10(PSD/(V^2/Hz))
Frequency (GHz)
3D infinite loop array
2D infinite loop array
1D infinite loop array
0D active loaded loop
Log-Scale PSD
Frequency (GHz)
1D
1D array (L_L=-20nH)
1D array (L_L=-30nH)
1D array (L_L=-40nH)
NF (dB)
Frequency (GHz)
1D infinite loop array(5mm)
1D infinite loop array(10mm)
1D infinite loop array(20mm)
0D passive loaded loop
log10(PSD/(V^2/Hz))
Frequency (GHz)
3D infinite loop array
2D infinite loop array
1D infinite loop array
0D passive loaded loop
Log-Scale PSD
Frequency (GHz)

Conclusions



- Negative impedance loads can impr
  - Antenna impedance bandwidth
  - AMC bandwidth
  - Bulk material properties
- Stability can be maintained
  - Use Nyquist criterion or FDTD
  - Parasitics are VERY important.
- Can improve SNR through design
  - Gain control → NF control…
- Improvement over passives?
- Increase NFC frequency? (currently 1GHz)
- Increase NFC power-handling?